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PEER REVIEWED ARTICLE

A comparison of mortality transition in China and India, 1950–2021

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Abstract

This paper compares mortality transition in China and India in the period 1950–2021, highlighting similarities and differences. Mortality transition has been inconsistent in both countries, but differences remain. In China, the transition has been spread evenly across the age range, while in India, it has primarily been confined to younger ages, being markedly slow in ages 35–90. This difference in the older ages appears to explain the main difference between the respective mortality transition in the two countries. To address its ongoing mortality transition, the paper concludes, India needs to reinvigorate its health-care delivery system to meet the health care needs of the old people. The paper also emphasises using geometric mean of the age-specific probabilities of death as an appropriate measure to analyse mortality transition.

Keywords: mortality transition, China, India, geometric mean, life expectancy, decomposition.

Introduction

China and India are the only billion-plus countries in the world and, together, they accounted for almost 36 per cent of the world's population in 2021 (United Nations, 2022). The world's demographic prospects have therefore been, and

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continue to be, closely linked to the demographic transition of both countries. The comparative demography and development has in both cases always been of interest to both demographers and development experts (Coale, 1983; Adlakha and Banister, 1995; Dummer and Cook, 2008; Singh and Liu, 2012; Golley and Tyres, 2013; Joe et al, 2015; Chaurasia, 2017; 2020). While around 1950, China and India were at a very similar stage of demography and development, the situation has now changed radically. China is now at a very advanced stage of demographic transition and its population has just started to decline. In contrast, India continues to be in the middle of a transition, although it has recently achieved replacement fertility. The population of India continues to increase, albeit at a slower pace, and its population now appears to have surpassed that of China.

Socially, culturally and politically, China and India are poles apart. This has implications for both the population and the development processes in the two countries. In China, the Han ethnic community constitutes more than 90 per cent of the population (Chen et al., 2009). China is also one of the few countries of the world that has never been entirely colonised by foreign powers, with the result that its society, culture and economy – especially of the mainland – has remained largely unaffected by colonisation. After becoming communist in 1948, the country adopted a single-party political system which has left virtually no scope for democratic influence on government policies and programmes with implications for demography and development.

The social, cultural and political diversity of India, on the other hand, is so great that India is often called 'the country of countries'. Its society and culture are both deeply complex and fragmented, and one reason is the rule by foreign invaders that lasted for almost 1,000 years. Indian society is broadly divided into two classes – the rulers and the ruled – and there is a great divide between the two. After independence in 1947, India adopted a multi-party political system which has led to a democratic diversity of the extreme order. One implication of this political system is that there has rarely been political consensus on key issues related to demography and development. This lack of political consensus has influenced India's demographic processes as well as its social and economic development.

It is against the above background that this paper analyses the mortality transition in China and India in 1950–2021 through a comparative perspective.

This comparison is important since mortality transition signals the beginning of demographic transition. Falling mortality has also been suggested as an indicator of economic success. Moreover, transition in mortality can also throw light on the transition in social and economic development processes in terms of social inequalities, including gender and racial disparities (Sen, 1998). Understanding mortality transition is therefore the first step towards understanding demographic transition and towards characterising social and economic development processes. The analysis in this paper shows that China's mortality transition has been different from that in India during the investigated period. The analysis also shows that mortality transition described by the time trend in different measures of aggregate mortality are essentially different.

Mortality transition encompasses a decrease in aggregate mortality level and change in the age pattern of mortality. The most commonly used measure of analysing mortality transition at the aggregate level is the life expectancy at birth (e_0), which is independent of population age structure so that it can be compared over time and across different populations at different stages of mortality transition. However, e_0 reflects mortality experience of a hypothetical population and not of the real population. It is the average of the age distribution of deaths and is therefore not unique. Different age distributions of deaths may have the same e_0 (Goerlich Gisbert, 2020). The increase in e_0 is also influenced more by the decrease in the risk of death at older ages than the decrease in the risk of death at younger ages (Chaurasia, 2023; Keyfitz, 1977; Vaupel, 1986).

In view of the limitations of e_0, alternative measures of aggregate mortality have been suggested. One alternative is the median age at death (INE, 1952, 1958) while the other is the modal age at death (Canudas-Romo, 2008). The geometric mean of age-specific death rates (Schoen, 1970) and geometric mean of age distribution of deaths (Ghislandi et al., 2019) have also been suggested. Goerlich Gisbert (2020) has suggested a distributionally adjusted e_0 that considers not only the level but also the age distribution of deaths. Chaurasia (2023) has used the geometric mean of the age-specific probabilities of death as the measure of aggregate level of mortality to analyse mortality transition in India. The advantage of using the probability of death rather than death rate is the ease in the interpretation of the probability of death, since the conditional probability of death is defined as the number of deaths over the course of a given time period and in an age cohort divided by the living population at the start of the time period (King and Soneji, 2011). The probability of death always ranges between 0 and 1 and is used for the construction of the life table and calculation of e_0 (de Beer 2012).

The paper is organised as follows. The next section of the paper describes the data source and analytical methods used in the paper. The third section analyses mortality transition in China and India between 1950 and 2021, in terms of the time trend in the two measures of aggregate mortality – the life expectancy at birth (e_0) and the geometric mean of the age-specific probabilities of death. The fourth section of the paper analyses how changes in the age-specific probabilities of death contribute to the change in the geometric mean of age-specific probabilities of death. The fifth section analyses the transition in age-specific probabilities of death in the two countries by fitting a non-parametric model. The sixth section decomposes the difference in the age-specific probabilities of death between the two countries into four components, the difference in average probability of death across all years and all ages, difference in average probability of death across all ages in different years, difference in average probability of death across years in different ages and difference in a residual component that is not explained by the first three components. The last section of the paper summarises main findings of the analysis that characterise the difference in mortality transition between China and India since 1950.

Data and methods

This paper is based on the United Nations Population Division's (UNPD) estimates of the age-specific probabilities of death by single years of age for the period 1950–2021, part of its 2022 revision of the world population prospects (United Nations, 2022). These estimates are based on a uniform methodology and a standard set of assumptions for all countries and they therefore permit inter-country comparison. This may not be the case with the official estimates, as the official estimates of different countries may be based on different methodologies and different sets of assumptions. The difference between the United Nations' estimates and the official estimates of the age-specific probabilities of death is, however, small in both China and India.

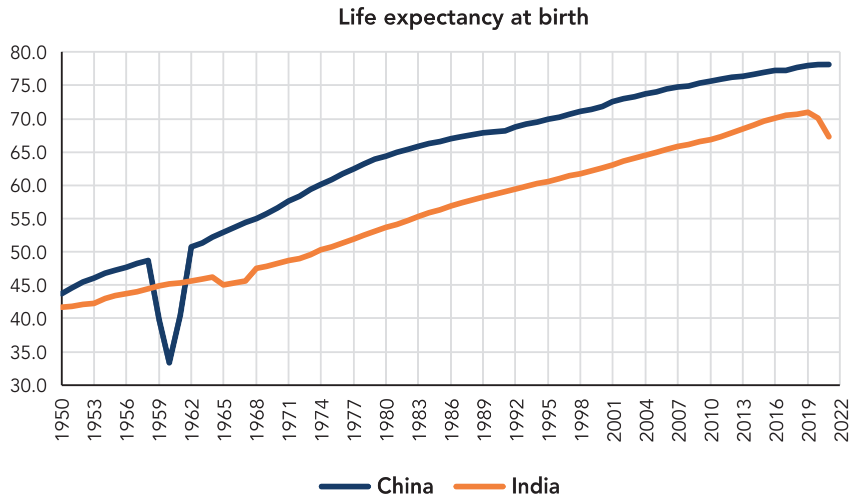
The methods used for analysing transition in mortality in the two countries involve analysis of the time trend and the decomposition analysis. The appendix to the paper describes, in detail, the approach adopted for the analysis of the time trend and the method of decomposition. The analysis of the time trend is based on the underlying assumption that the trend may be different in different time-segments of the trend period 1950–2021. On the other hand, the change in the aggregate measure of mortality has been decomposed into the change in different components of the aggregate measure to understand the determinants of change.

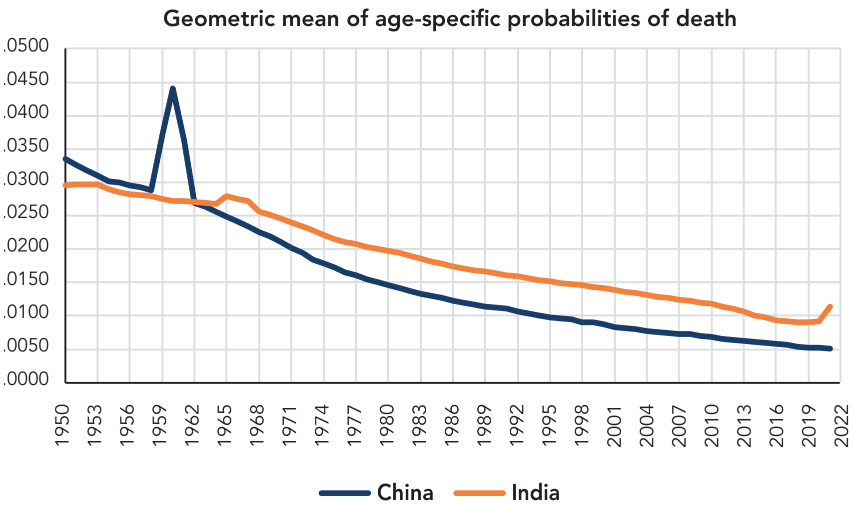
Trend in aggregate measures of mortality

Estimates prepared by the United Nations suggest that life expectancy at birth (e_0) in China increased from around 43.7 years in 1950 to more than 78 years in 2021 (United Nations, 2022), an increase of more than 34 years between 1950 and 2021 (Figure 1). In India, e_0 increased by around 25 years during this period, from 41.7 years in 1950 to 70.9 years in 2019, but then decreased to 67.2 years in 2021 possibly because of the mortality impact of COVID-19 pandemic. Similarly, the geometric mean of age-specific probabilities of death decreased from 0.0234 in 1950 to 0.0048 in 2021 in China, whereas in India, it decreased from 0.0286 in 1950 to 0.0086 in 2019 and then increased to 0.0109 in 2021. In China, e_0 increased while the geometric mean of age-specific probabilities of death decreased even during the COVID-19 pandemic. In contrast, e_0 in India decreased, while the geometric mean of age-specific probabilities of death increased during the pandemic. Overall, the geometric mean of age-specific probabilities of death was lower in India than in China in the period 1950–1961 while, except the short period between 1959 and 1961, China's e_0 has always been higher than that of India.

Figure 1 also suggests that, in both countries, the trend in e_0 and in the geometric mean of the age-specific probabilities of death changed many times between 1950 and 2021. I have, therefore, analysed the trend using the joinpoint regression model, which first identifies inflexion point(s) in the trend and then estimates the trend between two successive inflexion points assuming a linear trend. If there is no point of inflexion, the joinpoint regression model reduces to a simple linear model. I have used the Joinpoint Regression Program software (National Cancer Institute, 2023) for this purpose. The software requires, a priori, minimum and maximum number of joinpoints. When the number of joinpoints is zero, the software fits a straight line. The software provides estimates of annual per cent change (APC) in different time-segments of the trend period. The APCs in different time-segments may be combined into average annual per cent change (AAPC) during the trend period as the weighted average of APC in different time-segments with weights equal the length of the time-segment. The AAPC gives

Figure 1: Trend in summary measures of mortality in China and India, 1950–2021





SOURCE: AUTHOR

a better reflection of the trend over time compared to the conventional rate of change obtained through the application of the linear regression analysis on a logarithmic scale (Clegg et al., 2009).

Table 1 presents results of the joinpoint regression analysis. In China, the trend in e_0 changed four times between 1950 and 2021 and the trend has been different in different time-segments. Combining the APC in different time-segments, the average annual per cent change (AAPC) in e_0 in China was 0.849 per cent per year in 1950–2021 with the rate of increase slowing down considerably after 1981. In India, the trend in e_0 changed five times. In 1963–1966, e_0 in the country remained virtually stagnant. The rate of improvement in e_0 in India was slower than that in China before 1986, but faster than China between 1986 and 2019. The gap in e_0 between the two countries, therefore, first increased and then decreased to the lowest since 1965 in 2017. After 2017, the gap increased again and, during the COVID-19 pandemic (2020–2021), e_0 in India decreased very rapidly leading to a very rapid increase in the gap. The AAPC in e_0 in India during 1950 and 2021 has also been much slower than that in China.

Table 1. Analysis of the trend in e_0 in China and India, 1950–2021

Segment	Time-segment		Annual per cent change			Test statistic (t)	Prob > |t|
	Lower	Upper	Estimate	Lower CI	Upper CI		
	China						
1	1950	1957	1.909	1.706	2.112	18.986	< 0.001
2	1957	1960	-10.563	-12.032	-9.069	-13.491	< 0.001
3	1960	1963	13.650	11.783	15.548	15.462	< 0.001
4	1963	1981	1.261	1.209	1.313	49.180	< 0.001
5	1981	2021	0.484	0.470	0.498	69.032	< 0.001
Full Range	1950	2021	0.849	0.748	0.949	16.575	< 0.001
	India						
1	1950	1963	0.827	0.794	0.861	49.811	< 0.001
2	1963	1966	-0.801	-1.573	-0.022	-2.060	0.044

Segment	Time-segment		Annual per cent change			Test statistic (t)	Prob > |t|
	Lower	Upper	Estimate	Lower CI	Upper CI		
3	1966	1969	1.921	1.127	2.721	4.875	< 0.001
4	1969	1986	1.047	1.023	1.071	86.807	< 0.001
5	1986	2019	0.683	0.674	0.691	162.607	< 0.001
6	2019	2021	-2.639	-3.405	-1.867	-6.786	< 0.001
Full Range	1950	2021	0.690	0.638	0.742	26.203	< 0.001

SOURCE: AUTHOR

The trend in the geometric mean of the age-specific probabilities of death, on the other hand, changed five times in both countries during the period 1950–2021 (Table 2). The points of inflexion in the trend in the geometric mean of age-specific probabilities of death have been the same as e_0 during 1950–1963 in China and during 1950–1966 in India. However, after 1963 in China and after 1966 in India, the points of inflexion in the trend in the geometric mean of the age-specific probabilities of death have been different from the points of inflexions in the trend in e_0. A comparison of tables 1 and 2 suggests that the mortality transition reflected by the trend in e_0 is different from the mortality transition reflected by the trend in geometric mean of age-specific probabilities of death in both countries. One reason for this difference is that the trend in e_0 depicts mortality transition in a hypothetical population, whereas the trend in geometric mean of age-specific probabilities of death depicts mortality transition in the real population. A comparison of period age-specific probabilities of death in 1950 with those for the cohort born in 1950 for ages 0–71 years reveals that the two sets of age-specific probabilities of death are different in both countries. For example, a person born in 1950 in China was 71 years old in 2021 and the probability of death for the person in the 71st year of life was 0.0241, whereas the probability of death in 71 years of age in 1950 was 0.0945. The e_0 for the year 1950 is calculated assuming that a person born in 1950 will be subject to age-specific probabilities of death that prevailed in the year 1950. However, the actual age-specific probabilities of death to which a person born in 1950 was subjected to the 1950 cohort age-specific probabilities of death were substantially lower than the age-specific probabilities that prevailed in 1950. Obviously, the actual age-specific risk of death experienced by a person born in 1950 is different from the age-specific risk of death reflected by the age-specific probabilities of death that prevailed in the country in 1950. In India also, the risk of death experienced by a person born in 1950 in the 71st year of life was different from the probability of death in 71 years of age in the year 1950, although the difference between the cohort and the period age-specific probabilities of death in India is relatively narrower than that in China.

The difference in the trend in e_0 and the trend in the geometric mean of the age-specific probabilities of death suggests that it is more appropriate to use the geometric mean of the age-specific probabilities of death as the summary measure of mortality than the life expectancy at birth. The use of geometric mean of age-specific probabilities of death as the summary measure of mortality has many advantages. The geometric mean gives equal weight to probabilities of death in different ages. This is not the case with e_0. The change in any of the age-specific probabilities of death results in a change in the geometric mean of the age-specific probabilities of death, which is not the case with the median or the mode of the age-specific probabilities of death. The geometric mean also addresses the problem of perfect substitutability associated with arithmetic mean.

Table 2: Analysis of the trend in the geometric mean of age-specific probabilities of death (*g*) in China and India, 1950–2021

Segment	Time-segment		Annual per cent change			Test statistic (t)	Prob > |t|
	Lower	Upper	Estimate	Lower CI	Upper CI		
	China						
1	1950	1957	-2.494	-2.814	-2.172	-15.388	< 0.001
2	1957	1960	14.364	11.010	17.819	9.036	< 0.001
3	1960	1963	-15.646	-18.120	-13.097	-11.455	< 0.001
4	1963	1966	-1.100	-4.000	1.889	-0.744	0.460
5	1966	1979	-3.753	-3.889	-3.618	-54.330	< 0.001
6	1979	2021	-2.516	-2.536	-2.496	-243.490	< 0.001
Full Range	1950	2021	-2.620	-2.832	-2.409	-23.959	< 0.001

Segment	Time-segment		Annual per cent change			Test statistic (t)	Prob > |t|
	Lower	Upper	Estimate	Lower CI	Upper CI		
	India						
1	1950	1963	-0.873	-1.005	-0.740	-13.161	< 0.001
2	1963	1966	1.035	-1.741	3.890	0.741	0.462
3	1966	1976	-2.741	-2.968	-2.513	-23.807	< 0.001
4	1976	2009	-1.650	-1.683	-1.616	-97.609	< 0.001
5	2009	2019	-3.269	-3.495	-3.043	-28.475	< 0.001
6	2019	2021	12.909	9.802	16.103	8.720	< 0.001
Full Range	1950	2021	-1.398	-1.544	-1.251	-18.578	< 0.001

SOURCE: AUTHOR

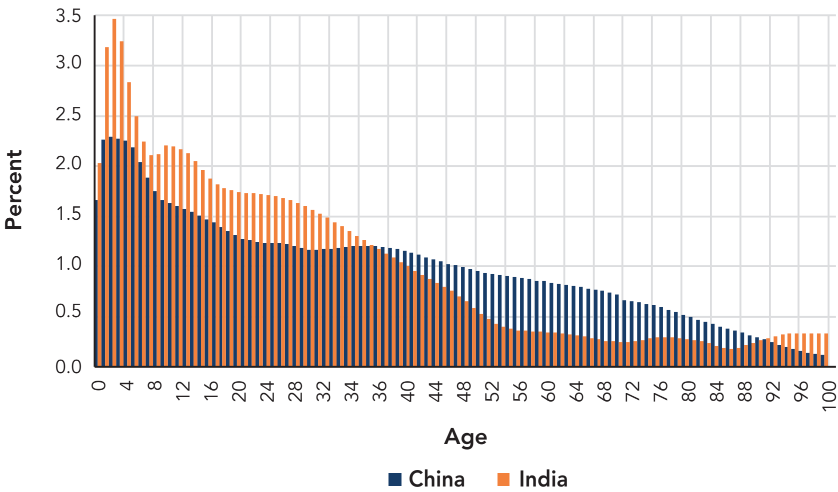
I have used the age-specific probabilities of death, instead of the age-specific death rates, to analyse mortality transition by age. The reason is that the probability of death in the last, open-ended age interval is always equal to 1 so that the geometric mean of the age-specific probabilities of death is not influenced by the risk or the probability of death in the last, open-ended age interval. This is not the case with the death rate and the difficulty in estimating the death rate in the last, open-ended age interval is well-known. The death rate in the last, open-ended age interval is always an approximation. It is also straightforward to decompose the change in the geometric mean of the age-specific probabilities of death to the change in the probability of death in different ages. This decomposition helps in characterising and comparing mortality transition.

Decomposition of the change in g

I have used index decomposition analysis (IDA) approach to decompose the change in the geometric mean of age-specific probabilities of death (g) over time into the change in the age-specific probabilities of death. The IDA approach was first used in the early 1980s to decompose the change in the industrial energy consumption and has since been widely applied in energy and emission studies (Ang, 2015). Among different IDA approaches, the Logarithmic Mean Divisia Index (LMDI) approach is the most popular (Ang, 2005; Ang and Liu, 2001) because of many desirable properties it possesses (Ang, 2004). The approach has been used in analysing the contribution of different factors to the increase in energy consumption and Carbon Dioxide emission (Makutėnienė et al, 2022; Lisaba and Lopez, 2020; He and Myers, 2021; Tu et al., 2019). It has also been used in analysing how the change in different factors contribute to the change in demographic indicators (Chaurasia, 2023). The details of the LMDI approach of decomposition are given in the appendix.

Results of the decomposition analysis are presented in Figure 2 and summarised in Table 3. More than 53 per cent of the decrease in *g* in China during 1950–2021 is attributed to the decrease in the probability of death in the population younger than 35 years of age, whereas this proportion is almost 70 per cent in India. The decrease in the probability of death in the population aged 35–90 accounts for a decrease of almost 45 per cent in *g* in China but less than 28 per cent in India. The contribution of the decrease in the probability of death in the age group 55–90 years was less than 10 per cent in India but more than 23 per cent in China. In both countries, decrease in the probability of death in the age group 1–14 years accounted for most of the decrease in *g* – around 24 per cent in China but almost 31 per cent in India.

Figure 2: Proportionate contribution of the decrease in the probability of death in different ages to the decrease in *g* in China and India, 1950–2021



SOURCE: AUTHOR

Table 3. Contribution of the change in probability of death in different age-groups to the change in the geometric mean of the age-specific probabilities of death (*g*) in China and India, 1950–2021

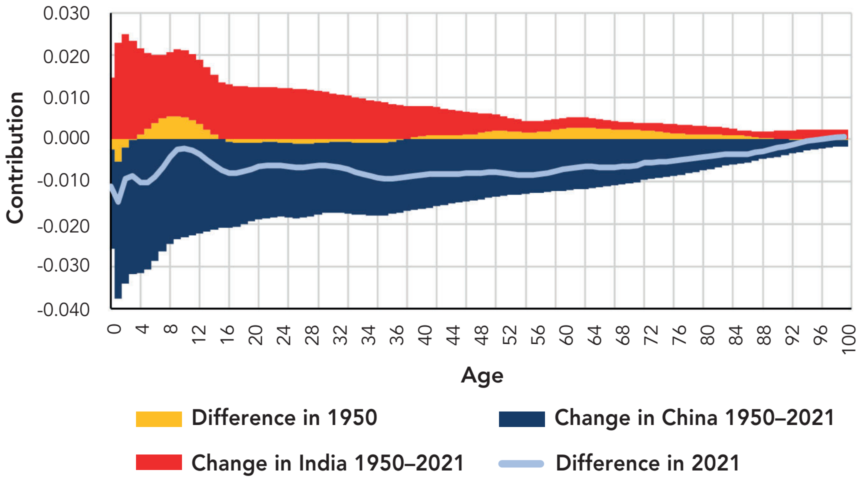
Age	China						
	Change in *g* during the period						
	1950–2021	1950–1957	1957–1960	1960–1963	1963–1966	1966–1979	1979–2021
	-0.0127	-0.0042	0.0145	-0.0172	-0.0022	-0.0088	-0.0096
	Contribution of the change in the probability of death in the age group						
0	1.65	1.16	-1.36	1.36	1.05	1.25	1.96
1–4	9.02	7.69	-8.88	8.87	6.71	7.99	9.84
5–9	9.47	9.86	-9.97	9.98	6.94	7.83	10.30
10–14	7.87	10.74	-9.57	9.57	6.34	5.95	8.30
15–19	6.98	9.76	-6.85	7.48	6.92	5.85	6.84
20–24	6.28	8.56	-5.78	6.53	7.57	6.14	5.64
25–29	6.06	8.68	-5.64	6.07	6.96	6.69	5.21
30–34	5.95	9.34	-5.81	6.12	6.07	6.41	5.17
35–39	6.05	8.40	-5.71	6.26	5.95	6.08	5.52
40–44	5.78	6.96	-5.09	5.65	6.87	5.58	5.43
45–49	5.25	5.85	-5.09	5.31	6.49	5.60	4.83
50–54	4.80	5.55	-4.65	4.67	5.55	5.60	4.29
55–59	4.48	5.87	-4.65	4.52	4.85	5.13	4.04
60–64	4.21	4.56	-3.79	3.96	4.38	4.37	4.04
65–69	3.90	3.28	-3.64	3.64	4.92	3.81	3.96
70–74	3.41	1.72	-3.46	3.23	4.12	3.68	3.55
75–79	2.93	1.47	-3.43	2.84	3.37	3.55	3.04
80–94	2.33	-0.33	-2.30	1.87	2.06	3.11	2.56
85–89	1.77	-1.57	-1.85	1.21	2.08	2.34	2.21
90–94	1.17	-3.33	-1.42	0.57	0.86	1.81	1.87
95–99	0.67	-4.22	-1.05	0.32	-0.05	1.22	1.42

	India						
	Change in *g* during the period						
	1950–2021	1950–1963	1963–1966	1966–1976	1976–2009	2009–2019	2019–2021
	-0.0177	-0.0026	0.0006	-0.0063	-0.0088	-0.0029	0.0023
	Contribution of the change in the probability of death in the age group						
0	2.03	1.77	-0.55	0.70	1.75	1.81	0.38
1–4	12.72	11.45	-8.41	4.33	11.36	11.17	2.13
5–9	11.18	7.02	-30.79	2.14	11.11	11.98	1.99
10–14	10.49	7.31	-16.48	7.41	9.60	7.31	1.00
15–19	8.96	5.45	3.16	6.23	8.27	8.11	-2.95
20–24	8.58	5.45	4.83	6.04	7.63	8.99	-4.02
25–29	8.14	5.47	5.22	7.24	7.50	6.58	-3.93
30–34	7.19	5.77	3.17	8.21	6.28	5.60	-4.66
35–39	6.08	5.77	2.28	8.01	5.80	3.61	-5.28
40–44	4.99	5.51	-0.08	6.49	5.99	1.70	-5.75
45–49	3.96	5.42	-3.82	4.95	4.68	3.56	-6.91
50–54	2.67	5.35	-6.51	3.43	4.81	1.02	-7.31
55–59	1.84	5.21	-7.95	2.18	4.81	-0.54	-7.14
60–64	1.69	4.96	-8.88	1.55	3.74	2.48	-8.03
65–69	1.43	4.58	-9.08	2.17	2.74	3.01	-7.89
70–74	1.26	4.07	-8.75	2.92	1.96	3.16	-7.56
75–79	1.43	3.37	-7.11	3.46	1.96	2.90	-7.03
80–94	1.24	2.58	-4.33	4.44	0.95	4.05	-7.91
85–89	0.99	1.84	-2.97	5.37	0.19	4.55	-8.61
90–94	1.49	1.04	-1.76	6.34	-0.35	4.53	-6.19
95–99	1.64	0.62	-1.20	6.37	-0.76	4.41	-4.33

SOURCE: AUTHOR

Table 3 also decomposes the decrease in *g* in different time-segments in which the trend in *g* has been different. In China, the probability of death in the population aged at least 80 years increased during 1950–1957 and therefore contributed to increase, instead of decrease, in *g*. During 1957–1960, increase in the probability of death in all ages contributed to an increase in *g* in China and, after 1960, the decrease in the probability of death in all ages contributed to the decrease in *g* with the only exception of the population aged at least 90 years during 1963–1966. In India, decrease in *g* during 1950–1963 was due to a decrease in the probability of death in all ages whereas the increase in *g* during 1963–1966 was due to the increase in the probability of death in the population below 15 years of age and in the population aged at least 40 years. During 1966–2019, the decrease in the probability of death in all ages contributed to the decrease in *g* in India except during 1966–2019. During the COVID-19 pandemic, *g* increased, and this increase was due to the increase in probability of death in ages 15 years and above.

Figure 3: Decomposition of the difference in *g* in China and India in 2021



SOURCE: AUTHOR

The difference in *g* between the two countries at time $t_2>t_1$ depends upon three factors: the difference in *g* between two populations at time t_1 and the change in *g* in the two countries between t_1 and t_2 (Andreev et al., 2002; Jdanov et al., 2017). The LMDI approach can also be used to carry out this decomposition. The details are given in the appendix and the decomposition results are presented in Figure 3. In 1950, *g* was higher in China because of higher probability of death in ages 3–15 and 40–91. The decrease in the probability of death in ages below 96 years has been more rapid in China but, in ages 96 and above, more rapid in India. The contribution of the decrease in the probability of death in ages 0–37 and in ages 91 and above to the decrease in *g* has been higher in India, but, in ages 38–90, it has been higher in China. The lower mortality in China in 2021 has been due to a relatively more rapid decrease in the probability of death in ages 0–95. However, the decrease in the probability of death in ages 96 and above has been more rapid in India, which has contributed to narrowing down the difference in mortality between China and India in 2021.

Modelling age-specific probability of death

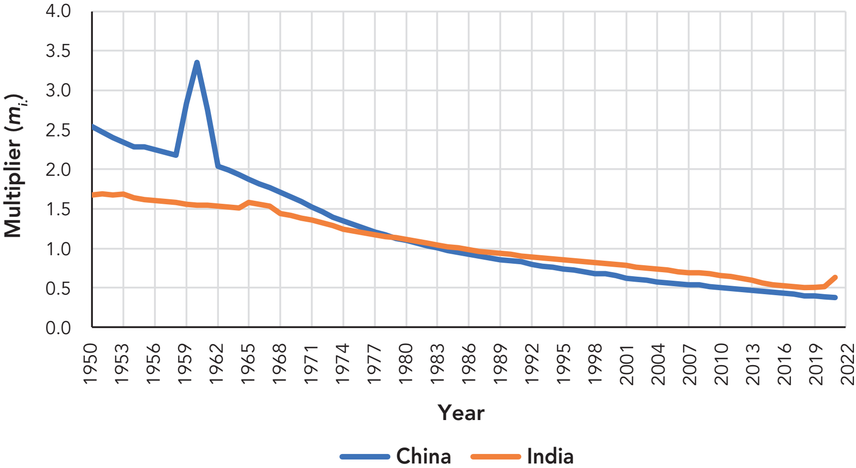
The age-specific probability of death in year *i* and age *j*, q_{ij} can be modelled in terms of: a common factor ($q_{..}$) across all *i* and *j*; a row factor or factor specific to the year ($q_{i.}$), which is common to all columns or ages *j* of the row or the year *i*; a column factor or factor specific to age ($q_{.j}$) which is common to all rows or years *i* of the column or age *j* and a residual factor r_{ij}, which is specific to each pair of *i* and *j* as shown in the appendix. This model can be fitted through the polishing technique, first proposed by Tukey (1977), by choosing an appropriate polishing function. The technique successively sweeps the polishing function out of rows (divides row values by the polishing function for the row), then sweeps the polishing function out of columns (divides column values by the polishing function for the column), then rows, then columns and so on, and accumulates them in 'all', 'row' and 'column' registers to obtain values of $q_{..}$, $m_{i.}$ and $m_{.j}$ respectively, and leaves behind a table of residuals (m_{ij}) which are specific to year *i* and age *j*. When the entire variation in q_{ij} is explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ or, equivalently, by $q_{..}$, $q_{i.}$ and $q_{.j}$, all m_{ij} are equal to 1. Otherwise m_{ij} reflects that part of q_{ij} that is not explained by $q_{..}$, $m_{i.}$ and $m_{.j}$. The mathematical formulation of the model is given in the appendix.

I have used the geometric mean as the polishing function to model q_{ij}. The use of geometric mean as the polishing function ensures that the geometric mean of residual multipliers m_{ij} is equal to 1; geometric mean of m_{i}, is equal to 1 and geometric mean of $m_{.j}$ is also equal to 1. It may be noticed that all the three multipliers $m_{i.}$, $m_{.j}$ and m_{ij} can be less than or more than 1. A value of the multiplier greater than 1 inflates $q_{..}$ whereas a value less than 1 deflates $q_{..}$. For example, if $m_{i.}$>1, then $q_{i.}$ is higher than $q_{..}$, but if $m_{i.}$<1, then $q_{i.}$ is lower than $q_{..}$ and $q_{i.}$ is equal to $q_{..}$ if $m_{i.}$=1. Similar interpretation can be made for the multiplier $m_{.j}$. On the other hand, if m_{ij}>1 than q_{ij} is higher than that determined by $q_{..}$, $m_{i.}$ and $m_{.j}$. If m_{ij}<1 than q_{ij} is lower than that determined by $q_{..}$, $m_{i.}$. When m_{ij}=1, q_{ij} is the same as that determined by $q_{..}$, $m_{i.}$ and $m_{.j}$.

For both countries, I have used 7100 q_{ij} values, *i* ranging from 1 (1950) to 71 (2021) and *j* ranging from age 0 to age 99 for modelling the age-specific probabilities of death in terms of the parameters $q_{..}$, $m_{i.}$, $m_{.j}$ and m_{ij}. The $q_{..}$ for China (0.0127) is estimated to be around 25 per cent lower than the $q_{..}$ for India (0.0170), which indicates that overall mortality level in India has been higher than that in China throughout the period under reference. If the period of the COVID-19 pandemic (2020–2021) is excluded from the modelling exercise, then $q_{..}$ is estimated to be around 32 per cent higher in India (0.0173) than that in China (0.0131). This implies that the COVID-19 pandemic has resulted in widening the difference in overall mortality between the two countries. The impact of the epidemic on mortality has been more in India than in China.

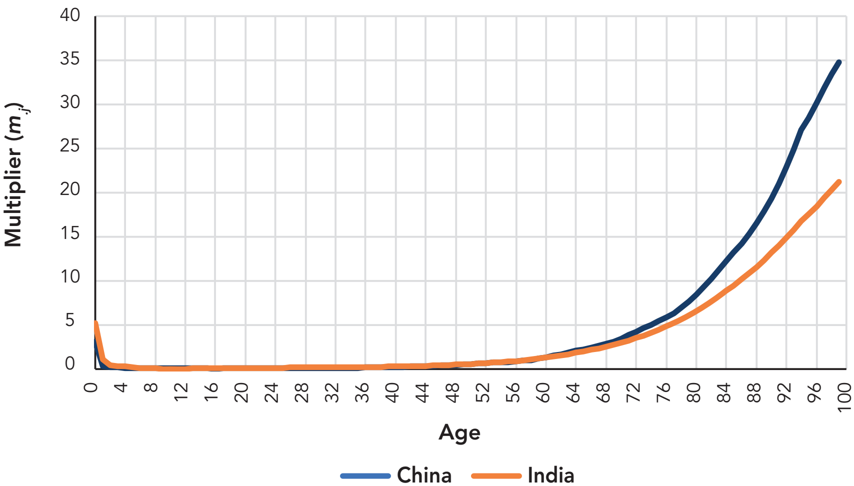
The modelling of the age-specific probabilities of death also reveals that the multiplier $m_{i.}$ has decreased in both countries during the period under reference, although the trend has been different in the two countries (Figure 4a). The joinpoint regression analysis suggests that $m_{i.}$ decreased in China at an average annual rate of decrease of 2.64 per cent per year during 1950–2021, whereas the average annual rate of decrease in India was only 1.41 per cent per year, which indicates that the decrease in the average mortality has been more rapid in China than in India. In China, $m_{i.}$ was greater than 1 up to 1983 but turned less than 1 after 1983. In India, $m_{i.}$ was greater than 1 up to 1985. An $m_{i.}$>1 implies $q_{i.}$>$q_{..}$. Figure 7 also shows that $m_{i.}$ was higher in China than in India during 1950–1978, but after 1978 it turned higher in India. The age multiplier $m_{.j}$ has also been different in the two countries (Figure 4b). The average probability of death in the first year of life during 1950–2021, $q_{.1}$, was more than 3 times $q_{..}$ in China but more than 5 times in India. However, in ages 8–13, multiplier $m_{.j}$ has been higher in China than in India, suggesting that, relative to $q_{..}$, the probability of death in China was higher than that in India in these ages. Similarly, in ages 60 and above, multiplier $m_{.j}$ was again higher in China than in India and the difference increased with age. The $q_{.90}$ was more than 19 times the $q_{..}$ in China but only about 13 times the $q_{..}$ in India. In ages 9–60, however, $q_{.j}$, relative to $q_{..}$ has been higher in India than in China.

Figure 4a: Trend in $m_{i.}$ in China and India, 1950–2021



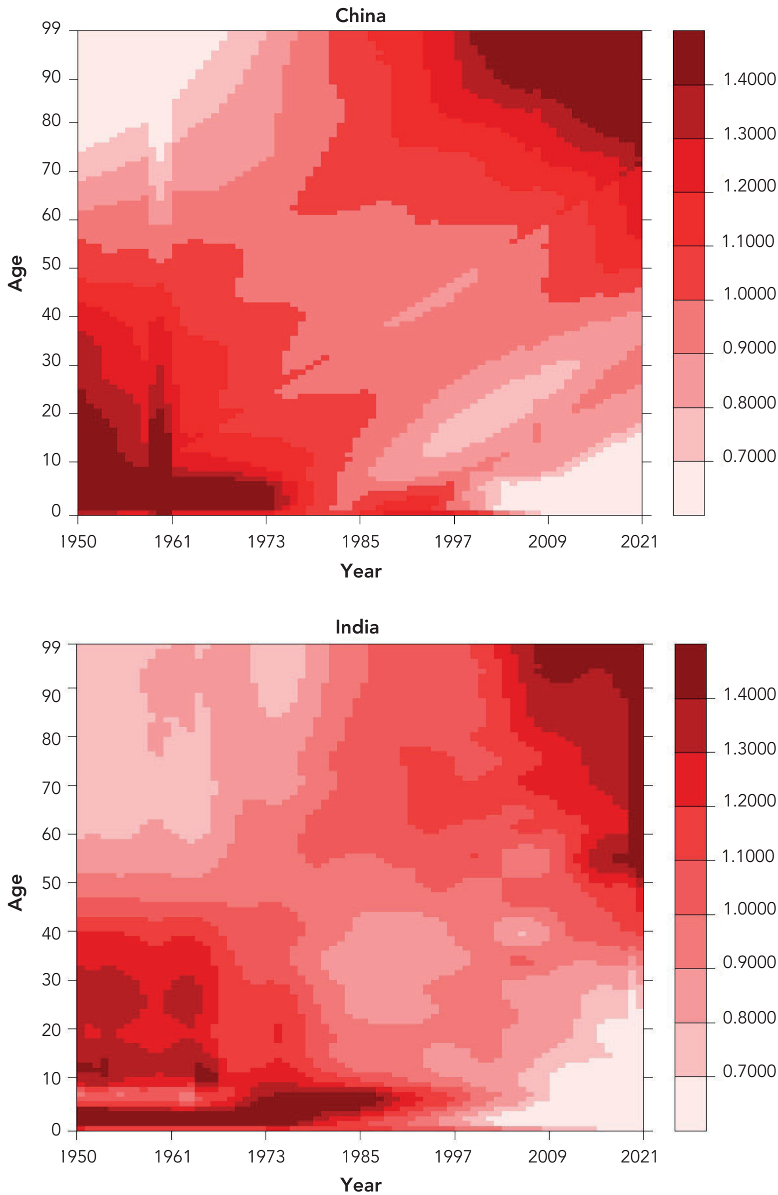
SOURCE: AUTHOR

Figure 4b: The age multiplier ($m_{.j}$) common to the period 1950–2021 in China and India



SOURCE: AUTHOR

Figure 5: Residual multipliers (m_{ij}) in China and India



SOURCE: AUTHOR

The trend in m_{ij} in the two countries is depicted in Figure 5. In both countries, m_{ij} decreased markedly with time in younger ages but increased in older ages, whereas the change in the middle ages has not been marked. An increase in m_{ij} implies an increase in actual probability of death specific to the year *i* and age *j* that is not explained by $q_{..}$, $q_{i.}$ and $q_{.j}$ and vice versa. For example, the probability of death in the first year of life in China was more than 30 per cent higher than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ in 1950 but more than 62 per cent lower in 2021. The probability of death in the first year of life remained higher than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ in China up to 2002. By contrast, actual probability of death in the first year of life in India was around 21 per cent higher than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ in 1950 but about 55 per cent lower in 2021. The actual probability of death in the first year of life remained higher than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ in India up to 1997. On the other hand, the actual probability of death at 80 years of age in China was around 24 per cent lower than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ in 1950 but was more than 59 per cent higher in 2021. Similarly, the actual probability of death at 80 years of age in India was around 38 per cent lower than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ in 1950 but was almost 54 per cent higher in 2021. In China, the actual probability of death in the year 1950 was higher than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ up to 56 years of age but up to 47 years age in 2021. In India, the actual probability of death in 1950 was higher than that explained by $q_{..}$, $m_{i.}$ and $m_{.j}$ up to 47 years of age but up to 37 years of age in 2021.

Decomposing the change in age-specific probabilities of death

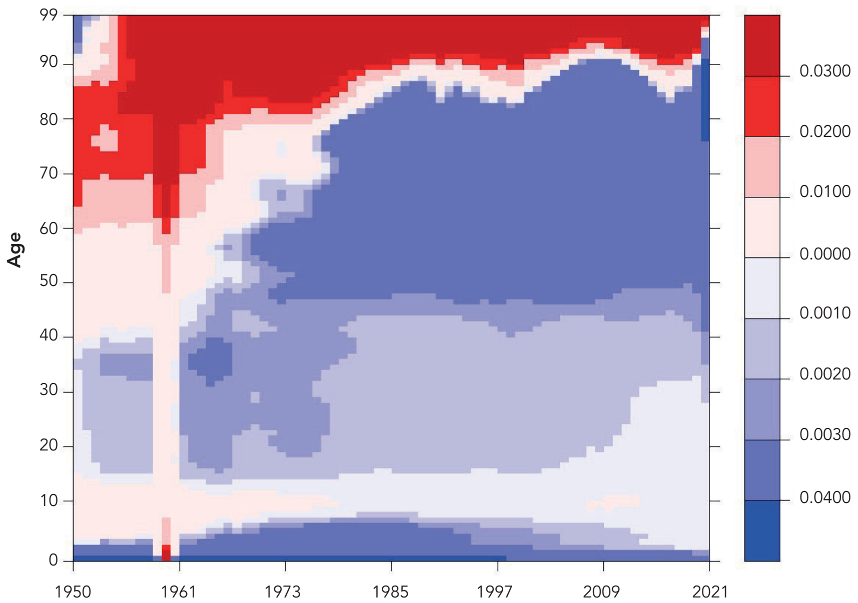
The modelling of q_{ij} in terms of $q_{..}$, $m_{i.}$, $m_{.j}$ and m_{ij} permits decomposing the difference in q_{ij} between China and India in terms of the difference in $q_{..}$, $m_{i.}$, $m_{.j}$ and m_{ij} following the LMDI approach. The decomposition results are presented in Figure 6 for the period 1950–2021 and for ages 0–99. A negative value of the difference means that q_{ij} is higher in India as compared to China. On the other hand, a positive value of the difference means a higher q_{ij} in China than in India. Figure 6 shows that q_{ij} was not always lower in China. The magnitude of the difference varied across ages and over time. In ages 50–90, the probability of death in India has markedly been higher than that in China after 1980, but in ages <5 years and ≥90 years, the probability of death has markedly been higher in China than in India.

Table 4. Decomposition of the difference in q_{ij} between China and India

Measure	1950			1985			2021		
	China	India	Difference/ contribution	China	India	Difference/ contribution	China	India	Difference/ contribution
Age 0 years									
q_{ij}	0.132	0.187	-0.049	0.041	0.102	-0.060	0.006	0.026	-0.020
$q_{..}$	0.013	0.017	-0.045	0.013	0.017	-0.019	0.013	0.017	-0.004
$m_{i.}$	2.545	1.678	0.065	0.952	1.003	-0.004	0.377	0.638	-0.007
$m_{.j}$	3.119	5.230	-0.080	3.119	5.230	-0.035	3.119	5.230	-0.007
m_{ij}	1.304	1.212	0.011	1.087	1.138	-0.003	0.378	0.448	-0.002
Age 40 years									
q_{ij}	0.012	0.012	0	0.003	0.005	-0.002	0.001	0.004	-0.003
$q_{..}$	0.013	0.017	-0.003	0.013	0.017	-0.001	0.013	0.017	-0.001
$m_{i.}$	2.545	1.003	0.005	0.952	1.003	0	0.377	0.638	-0.001
$m_{.j}$	3.119	5.230	-0.002	0.248	0.327	-0.001	0.248	0.327	0
m_{ij}	1.087	1.138	0	0.940	0.881	0	0.873	1.114	-0.001
Age 80 years									
q_{ij}	0.170	0.145	0.026	0.106	0.116	-0.010	0.063	0.111	-0.048
$q_{..}$	0.013	0.017	-0.046	0.013	0.017	-0.032	0.013	0.017	-0.025
$m_{i.}$	2.545	1.678	0.065	0.952	1.006	-0.006	0.377	0.638	-0.045
$m_{.j}$	8.548	6.611	0.039	8.468	6.611	0.027	8.468	6.611	0.021
m_{ij}	0.618	0.764	-0.033	1.033	1.026	0.001	1.560	1.537	0.001

SOURCE: AUTHOR

Figure 6: The difference in the age-specific probabilities of death (qij) between China and India, 1950–2021



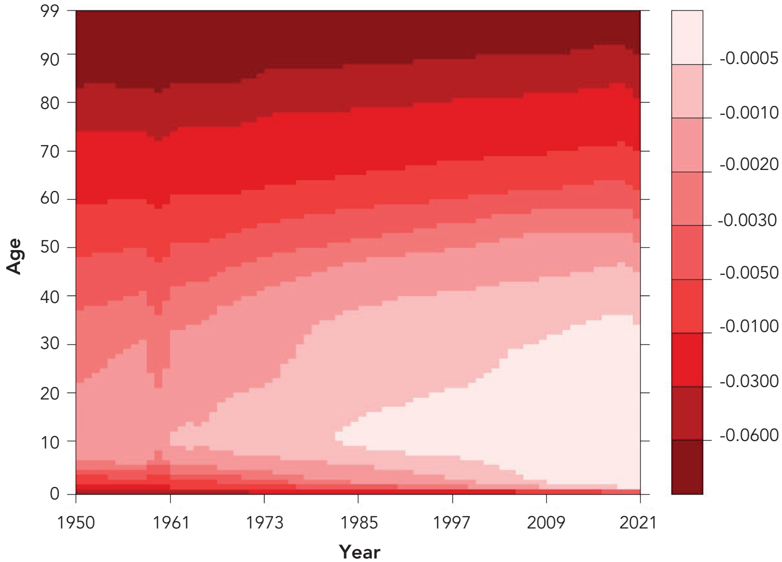
SOURCE: AUTHOR

The contribution of the difference in the four components – $q_{..}$, $m_{i.}$, $m_{.j}$ and m_{ij} – of q_{ij} to the difference in q_{ij} between China and India is summarised in Figures 7 through 10. The contribution of the difference in $q_{..}$ has always been negative as $q_{..}$ has always been lower in China than in India. However, this contribution increases with age, and has been the highest in the oldest ages both now and in the past. On the other hand, the contribution of the difference in $m_{i.}$, $m_{.j}$ and m_{ij} to the difference in q_{ij} between the two countries has been both negative and positive. The figures also suggest that there is a clear pattern in the contribution of the difference in $q_{..}$, $m_{i.}$ and $m_{.j}$, but the contribution of the difference in m_{ij} to the difference in q_{ij} has largely been different across time and age.

Table 4 shows how the difference in $q_{..}$, $m_{i.}$, $m_{.j}$ and m_{ij} has contributed to the difference in q_{ij} between China and India in selected years of life and in selected times. In the year 1950, the probability of death in the first year of life or in age

0 ($q_{1950,0}$) was 0.132 in China but 0.181 in India. This difference was due to higher overall mortality and higher age effect in India as the year or the time effect and the residual effect were lower in India than in China, which contributed to narrowing down the difference in the probability of death in the first year of life between the two countries in 1950. In 1985, $q_{1985,0}$ was 0.041 in China but 0.102 in India, and all the four components – $q_{..}$, $m_{i.}$, $m_{.j}$ and m_{ij} – contributed to lowering the probability of death in the first year of life in China as compared to that in India. A similar situation prevailed in 2021 when $q_{2021,0}$ was 0.006 in China but 0.026 in India. On the other hand, $q_{1950,40}$ was the same in the two countries because the negative difference in $q_{..}$ and $m_{.j}$ was offset by the positive difference in $m_{i.}$, but $q_{2021,40}$ was negative because $m_{i.}$ turned negative. Similar observations can be made about the difference in the probability of death at age 80.

Figure 7: Contribution of the difference in $q_{..}$ to the difference in q_{ij} between China and India

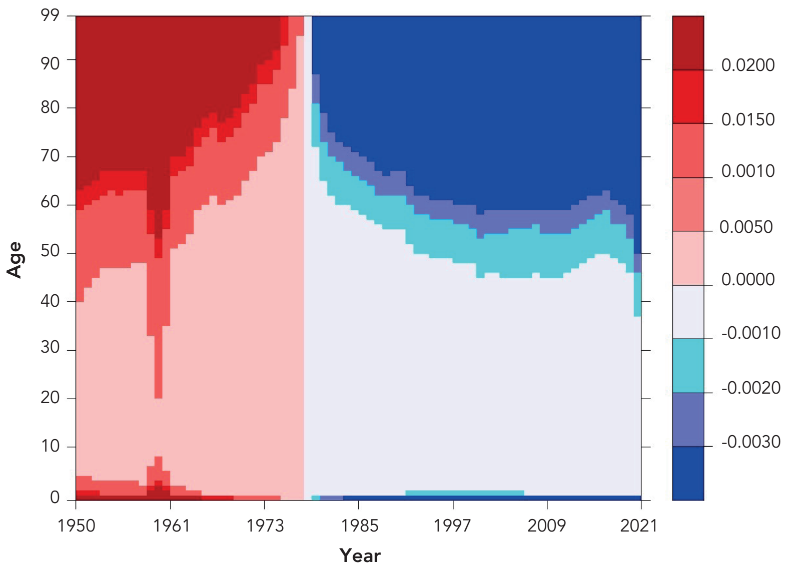


SOURCE: AUTHOR

Discussion and conclusions

This paper has highlighted the differences in mortality transition in China and India during the period 1950–2021. At the aggregate level, mortality transition has been more rapid in China. There are, however, ages at which the mortality transition in India has been more rapid relative to that in China. Mortality transition in China has not been limited to specific ages but has been spread across all ages up to 90 years of age. This has not been the case in India, where mortality transition has largely been confined to younger ages and there has been little transition in mortality in the ages 55–90. Mortality transition in younger ages, ages below 30 years, has been quite impressive in India, but the transition in younger ages in the country has been substantially compromised by very slow transition in older ages. The difference in mortality transition between China and India, then, is essentially located in the difference in mortality transition in older ages.

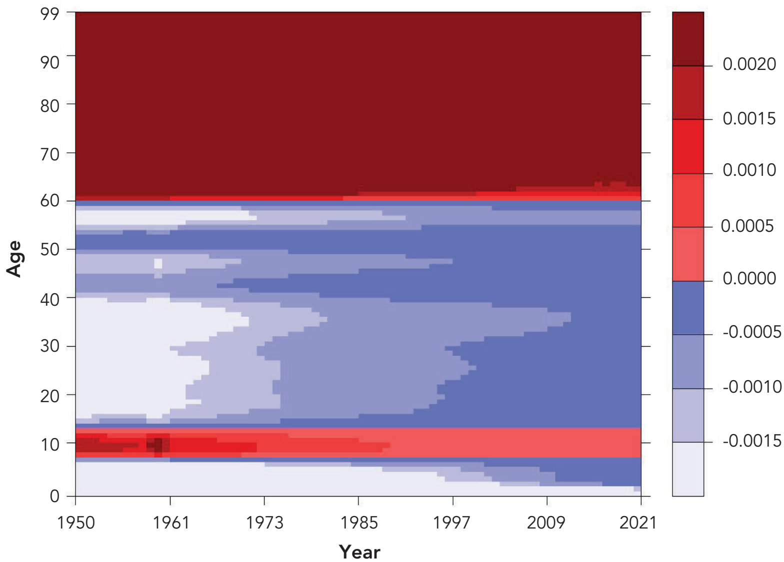
Figure 8: Contribution of the difference in $m_{i.}$ to the difference in q_{ij} between China and India



SOURCE: AUTHOR

The analysis also reveals that the impact of the COVID-19 pandemic on mortality has been much higher in India than that in China. This observation suggests that the health-care system in China, especially the public one, has been more efficient and effective in addressing the survival-related emergencies coming out of the pandemic than the health-care system in India. The almost universal coverage of China's public health-care system appears to have played a crucial role in preventing untimely deaths due to the pandemic. This does not appear to be the case with India. The life expectancy at birth in India is still around 70 years, which is low by international standards, while that in China compares with the life expectancy at birth in more developed countries of the world.

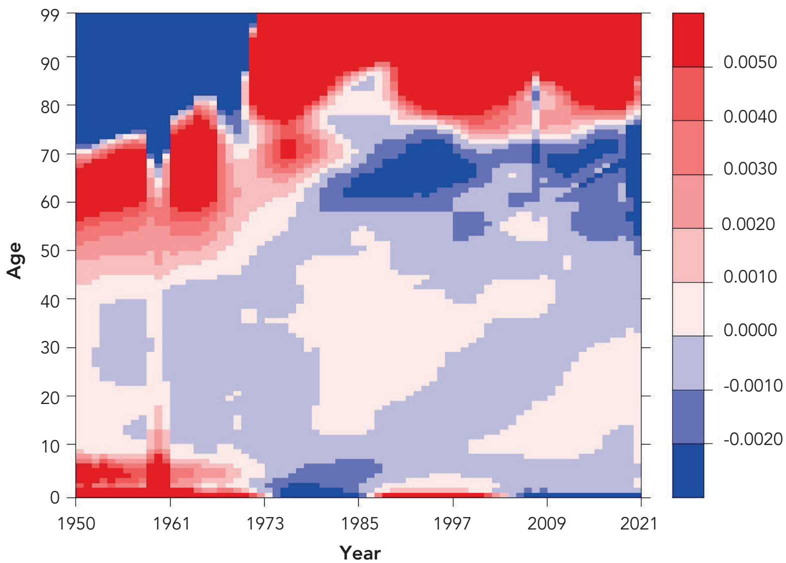
Figure 9: Contribution of the difference in $m_{.j}$ to the difference in q_{ij} between China and India



SOURCE: AUTHOR

India has now achieved replacement fertility, which means that an increasing proportion of the population of the country is getting older. The reduction in mortality in the younger ages will also hasten this process. The challenge for India, therefore, is to accelerate the reduction in mortality in older ages. This will require a reinvigoration of its health-care system, which has historically evolved following the extension approach of health services delivery, and it has primarily been directed towards addressing morbidity and mortality from infectious and communicable diseases through low-cost appropriate medical technology. This approach appears to have been successful in reducing the risk of death in younger ages, especially the risk of death during childhood. However, it has its limitations in addressing the health-care needs of the older population as non-communicable and degenerative diseases are the primary causes of morbidity and mortality in older ages. India needs an institution-based approach to meet the health-care needs of the old population, which will be increasing rapidly in the coming years.

Figure 10: Contribution of the difference in m_{ij} to difference in q_{ij} between China and India



SOURCE: AUTHOR

The present analysis also suggests that, at the aggregate level, mortality transition should be analysed in terms of the geometric mean of the age-specific probabilities of death and not in terms of the life expectancy at birth. Since the trend in the life expectancy at birth depicts mortality transition in a hypothetical population but not in the real population, as is the case with geometric mean of the age-specific probabilities of death. The trend in the life expectancy at birth is influenced by the age location of the mortality transition. If most of the mortality transition is confined to the younger ages of life, the improvement in the life expectancy at birth will be slower than when a mortality transition is well spread across all ages. In China, mortality transition has been fairly spread across the ages, whereas mortality transition in India has been confined largely to younger ages, and this appears to be a factor in the difference in the life expectancy at birth between the two countries. This weakness of the life expectancy at birth can be addressed by measuring mortality transition in terms of the geometric mean of the age-specific probabilities of death instead of the life expectancy at birth.

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APPENDIX

The change in the geometric mean of the age-specific probabilities of death (*g*) between two points in time, t_1 and t_2 ($t_2>t_1$), ∇g, can be written as:

$$\nabla g = g_2 - g_1 = \frac{g_2-g_1}{ln\left(\frac{g_2}{g_1}\right)} \times ln\left(\frac{g_2}{g_1}\right) = l_{21} \times ln\left(\frac{g_2}{g_1}\right)$$

(1)

where

$$l_{21} = \frac{g_2-g_1}{ln\left(\frac{g_2}{g_1}\right)} \quad (2)$$

is the logarithmic mean of g_2 and g_1 (Carlson, 1966, 1972; Bhatia, 2008; Ostle and Terwilliger, 1957; Lin, 1974). If q_i is the probability of death in age *i*, then

$$g = \left(\prod_{i=1}^{n} q_i\right)^{\frac{1}{n}}$$

(3)

$$ln\,ln\left(\frac{g_2}{g_1}\right) = ln\left(\frac{\left(\prod_{i=1}^{n} q_{2i}\right)^{\frac{1}{n}}}{\left(\prod_{i=1}^{n} q_{1i}\right)^{\frac{1}{n}}}\right)$$

(4)

$$ln\,ln\left(\frac{g_2}{g_1}\right) = \frac{1}{n}\sum_{i=1}^{n} ln\left(\frac{q_{2i}}{q_{1i}}\right)$$

(5)

Substituting,

$$\nabla g = g_2 - g_1 = \frac{l_{21}}{n} \times \sum_{i=1}^{n} ln\left(\frac{q_{2i}}{q_{1i}}\right)$$

(6)

Equation (6) decomposes the change in the geometric mean of the age-specific probabilities of death *g* into changes in the age-specific probabilities of death.

On the other hand, the difference in *g* between two populations, *A* and *B*, at time t_2 depends upon the difference in *g* between *A* and *B* at time t_1 and the difference in the change in g in A and B between t_1 and t_2 (Andreev et al., 2002; Jdanov et al., 2017). The difference in g between A and B at time t_2 may be written as:

$$\Delta g^2 = g_A^2 - g_B^2 = l_{AB}^2 \times ln\left(\frac{g_A^2}{g_B^2}\right) \tag{7}$$

where

$$l_{AB}^2 = \frac{g_A^2 - g_B^2}{ln\left(\frac{g_A^2}{g_B^2}\right)} \tag{8}$$

is the logarithmic mean of the g_A and g_B at time t_2. Now

$$ln\left(\frac{g_A^2}{g_B^2}\right) = ln\left(\frac{g_A^1}{g_B^1} \times \frac{g_B^1}{g_A^1} \times \frac{g_A^2}{g_B^2}\right) = ln\left(\frac{g_A^1}{g_B^1}\right) + ln\left(\frac{g_A^2}{g_A^1}\right) - ln\left(\frac{g_B^2}{g_B^1}\right) \tag{9}$$

Substituting in (8), I get

$$\Delta g^2 = g_A^2 - g_B^2 = l_{AB}^2 \times ln\left(\frac{g_A^1}{g_B^1}\right) + l_{AB}^2 \times ln\left(\frac{g_A^2}{g_A^1}\right) - l_{AB}^2 \times ln\left(\frac{g_B^2}{g_B^1}\right) \tag{10}$$

or

$$\Delta g^2 = g_A^2 - g_B^2 = \frac{l_{AB}^2}{n} \times \sum_{i=1}^{n} \quad ln\left(\frac{q_{1i}^A}{q_{1i}^B}\right) + \frac{l_{AB}^2}{n} \times \sum_{i=1}^{n} \quad ln\left(\frac{q_{2i}^A}{q_{1i}^A}\right) - \frac{l_{AB}^2}{n} \times \sum_{i=1}^{n} \quad ln\left(\frac{q_{2i}^B}{q_{1i}^B}\right) \tag{11}$$

Finally, the age-specific probability of death in the year i and age j, q_{ij}, can be modelled in terms of a common factor ($q_{..}$) across all i and j; a row- or year-specific factor ($q_{i.}$) which is common to all columns or ages, j, of the row or the year i; a column or age specific factor ($q_{.j}$) which is common to all rows or years, i, of the column or age, j, and a residual factor r_{ij}, which is specific to each pair of i and j as follows:

$$q_{ij} = q_{..} \times q_{i.} \times q_{.j} \times \frac{q_{ij}}{q_{..} \times q_{i.} \times q_{.j}} \tag{9}$$

or

$$q_{ij} = q_{..} \times \frac{q_{i.}}{q_{..}} \times \frac{q_{.j}}{q_{..}} \times \frac{q_{ij} \times q_{..} \times q_{..}}{q_{..} \times q_{i.} \times q_{.j}} \tag{10}$$

or

$$q_{ij} = q_{..} \times m_{i.} \times m_{.j} \times m_{ij} \tag{11}$$

where

$$m_{i.} = \frac{q_{i.}}{q_{..}} \tag{12}$$

$$m_{.j} = \frac{q_{.j}}{q_{..}} \tag{13}$$

$$m_{ij} = \frac{q_{ij} \times q_{..} \times q_{..}}{q_{..} \times q_{i.} \times q_{.j}} = \frac{\left(\frac{q_{ij}}{q_{..}}\right)}{\left(\frac{q_{i.}}{q_{..}} \times \frac{q_{i.}}{q_{..}}\right)} \tag{14}$$

Equation (11) suggests that difference in q_{ij} between two populations *A* and *B* can be decomposed into four components as follows:

$$\nabla q_{ij} = q_{ij}^A - q_{ij}^B = \left(q_{..}^A \times m_{i.}^A \times m_{.j}^A \times m_{ij}^A\right) - \left(q_{..}^B \times m_{i.}^B \times m_{.j}^B \times m_{ij}^B\right) \tag{15}$$

Now

$$\nabla q_{ij} = \frac{q_{ij}^A - q_{ij}^B}{ln\left(\frac{q_{ij}^A}{q_{ij}^B}\right)} \times ln\left(\frac{q_{ij}^A}{q_{ij}^B}\right) = \frac{q_{ij}^A - q_{ij}^B}{ln\left(\frac{q_{ij}^A}{q_{ij}^B}\right)} \times ln\left(\frac{q_{..}^A \times m_{i.}^A \times m_{.j}^A \times m_{ij}^A}{q_{..}^B \times m_{i.}^B \times m_{.j}^B \times m_{ij}^B}\right) \tag{16}$$

$$\nabla q_{ij} = \frac{q_{ij}^A - q_{ij}^B}{ln\left(\frac{q_{ij}^A}{q_{ij}^B}\right)} \times \left(ln\left(\frac{q_{..}^A}{q_{..}^B}\right) + ln\left(\frac{m_{i.}^A}{m_{i.}^B}\right) + ln\left(\frac{m_{.j}^A}{m_{.j}^B}\right) + ln\left(\frac{m_{ij}^A}{m_{ij}^B}\right)\right) \tag{17}$$

$$\Delta q_{ij} = \left\{\frac{q_{ij}^A - q_{ij}^B}{ln\left(\frac{q_{ij}^A}{q_{ij}^B}\right)} \times ln\left(\frac{q_{..}^A}{q_{..}^B}\right)\right\} + \left\{\frac{q_{ij}^A - q_{ij}^B}{ln\left(\frac{q_{ij}^A}{q_{ij}^B}\right)} \times ln\left(\frac{m_{i.}^A}{m_{i.}^B}\right)\right\} + \left\{\frac{q_{ij}^2 - q_{ij}^1}{ln\left(\frac{q_{ij}^2}{q_{ij}^1}\right)} \times ln\left(\frac{m_{.j}^2}{m_{.j}^1}\right)\right\} + \left\{\frac{q_{ij}^2 - q_{ij}^1}{ln\left(\frac{q_{ij}^2}{q_{ij}^1}\right)} \times ln\left(\frac{m_{ij}^2}{m_{ij}^1}\right)\right\}$$

$$\Delta q_{ij} = C_{..} + C_{i.} + C_{.j} + C_{ij} \tag{18}$$

$$C_{..} = \left\{\frac{q_{ij}^A - q_{ij}^B}{ln\left(\frac{q_{ij}^A}{q_{ij}^B}\right)} \times ln\left(\frac{q_{..}^A}{q_{..}^B}\right)\right\} \tag{19}$$

$$C_{i.} = \left\{ \frac{q_{ij}^{A} - q_{ij}^{B}}{ln\left(\frac{q_{ij}^{A}}{q_{ij}^{B}}\right)} \times ln\left(\frac{m_{i.}^{A}}{m_{i.}^{B}}\right) \right\} \quad (20)$$

$$C_{.j} = \left\{ \frac{q_{ij}^{2} - q_{ij}^{1}}{ln\left(\frac{q_{ij}^{2}}{q_{ij}^{1}}\right)} \times ln\left(\frac{m_{.j}^{2}}{m_{.j}^{1}}\right) \right\} \quad (21)$$

$$C_{ij} = \left\{ \frac{q_{ij}^{2} - q_{ij}^{1}}{ln\left(\frac{q_{ij}^{2}}{q_{ij}^{1}}\right)} \times ln\left(\frac{m_{ij}^{2}}{m_{ij}^{1}}\right) \right\} \quad (22)$$